A Synopsis on

"Contribution of Elzaki Transform to the Study of Nonlinear Partial Differential Equations"

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Title: Contribution of Elzaki Transform to the Study of Nonlinear Partial Differential Equations

1. Introduction:

The Nature phenomena is nonlinear, non deterministic in the field of Biology, Engineering, Oceanography and all Physical sciences such as propagation of sound, heat, electrostatic, electrodynamics, fluid or elasticity. Many processes or phenomena in physical sciences and biology are not continuous. Therefore these distinct physical phenomena can be formalized identically in terms of nonlinear partial differential equations. Hence it is important to develop efficient method to solve nonlinear partial differential equations. Therefore it is interest to study the nonlinear partial differential equations in day to day problems of the life. The literatures of nonlinear partial differential equations have been studied extensively by several authors in their views. Nonlinear partial differential equation constitutes a major and important topic that has been discussed under the heading of nonlinear analysis. It is known that every process of the universe involve some changes in the state and so it is in the model of the equation of certain type. Thus to every universal phenomena with uncertainties we have a nonlinear partial differential equations. The theory of nonlinear partial differential equation has attracted several mathematicians and at present study lies in the fact of physical or biological or social phenomena are nonlinear in nature. It can be modeled on ordinary and partial differential equation. Mathematicians solve the nonlinear partial differential equations by using different methods. Out of them, some Mathematicians was used different transform such as Laplace transform[1], Fourier transform [2], Hankel transform [3] etc. for solving nonlinear partial differential equations. In our research we will try to solve the nonlinear partial differential equations by using Elzaki transform.

Due to the simple formulation and useful properties of Elzaki transform. It has revealed that it can help to solve intricate problems in engineering Mathematics and applied science. However, despite the potential presented by this operator, only few theoretical investigations have appeared in the literature. Most of the available transform theory books, if not all, do not refer to Elzaki transform. On the other hand, for historical accountability, we must note that a related formulation, called s-multiplied Laplace transform, was announced as early as 1948 if not before, Tarig M. Elzaki and Salih M. Elzaki showed Elzaki transform applications to partial differential equations, ordinary differential equations, system of ordinary and partial differential equations and integral equations [4-9].

If the function f(t), $t \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of f(t) is given by

$$E\{f(t),v\} = T(v) = v\int_0^{\infty} e^{\frac{-t}{v}} f(t)dt, \quad v \in [-k_1,k_2] \text{ and } k_1,k_2 > 0$$

2. Review of Literature and development in the subject:

Many methods exists in the literature for finding approximate and exact solutions of partial differential and integral equations viz. Tanh Method [10], Laplace Decomposition Method (LDM) [11], Sine-Cosine Method [12], Inverse Scattering method [13], Hirota bilinear method [14], Exp-Function method [15], Backlund Transformation [16], Adomian Decomposition Method (ADM) [17] and so on.

Inokuti.et.al proposed a general Lagrange multiplier method to solve nonlinear problems in quantum mechanics [18]. Lagrange multiplier method is modified by He. J. M. in to an iteration method i.e. Variation Iteration Method [19]. G. Adomian started the study of nonlinear partial differential equations and he developed method, namely Adomian Decomposition Method (ADM) which is very useful [20]. M. A Mohamed, M. S Torky used the Laplace Decomposition Method (LDM) for numerical algorithm to solve nonlinear ordinary and partial differential equation [21]. Hany N. Hassan and Hassan K. Saleh used the Fourier Spectral Method for solving some nonlinear partial differential equations [22]. Tarig M. Elzaki find the solution of nonlinear partial differential equations by using the New Laplace Variational Iteration Method [23].

Tarig M. Elzaki showed that Elzaki transform can be effectively used to solve ordinary differential equations [4] and engineering control problems. Salih M. Elzaki presented applications to convolution type integral equations, production problems and Laplace –Elzaki duality was highlighted, and used to establish many fundamental useful properties [24]. Krima Patel use the Elzaki Transform Iterative Method for solving Fokker-Planck equation (FPE) which was first applied to investigate the Brownian motion of particles, is now largely employed in various generalized forms in Physics, Engineering biology and Chemistry [25]. Ekhlass S. Al-Rawi and Merna A. Samarchi implemented the Elzaki decomposition technique (EDM) to solve Volterra–Fredholm integro-differential equations of higher-order [26]. A. C. Loyinmi and T. K. Akinfe proposed an algorithm which involves coupling of Elzaki transform and homotopy perturbation method on the Burgers-Huxley equation which is a type of nonlinear advection-diffusion partial differential equation [27].

3. Objective of Research:

The present study will be a basic new and original contribution to the theory of nonlinear partial differential equations. In the present study we shall develop the modified hybrid Adomian method to existence the solution of nonlinear partial differential equations involving mixed partial derivatives with the help of Elzaki transforms and Adomian polynomial.The main objectives of the present study are as follows:

1. To discuss some known special kinds of nonlinear partial differential equations by using Adomian polynomial and Elzaki transform
2. To discuss the nonlinear partial differential equations for their different behavior of the solutions.
3. To find the exact solution of Camassa-Holm equation, Benjamin-Bona-Mahony Equation, and Modified Benjamin-Bona-Mahony Equation by using the combination of Elzaki transform.
4. To discuss some applications of the nonlinear partial differential equations studied in the present work.

4. Methodology:

1) Collection and study of some books and research papers related to the research topic.
2) Analysis of research papers and their literature review.
3) In literature, there are several methods for the solutionsof nonlinear partial differential equations such as Adomian Decomposition Method, Homotopy perturbation method, Parameter expanding method, Laplace substitution method and Iterative Method etc. We shall be adopting the Adomian decomposition method, Laplace substitution method and Elzaki transform for the study of nonlinear partial differential equations.
4) We will try to represents solutions of nonlinear partial differential equations graphically.
5) To learn software's like Mathematica, Mat-lab, Maxima and Python.
6) To learn LaTeX for scientific typing.

5. Importance of study:

Nowadays finding the solution of nonlinear partial differential equations has become of prime importance in the field of applied sciences. Obtaining exact or approximate solutions to nonlinear partial differential equations is still a significant problem and has wide research scope. However, finding solutions to these equations is a difficult task. So finding such solutions to non linear partial differential equations has great significance in modern science and engineering. We hope that the present

work will be useful for scientists, engineers, researchers, mathematicians at national and international level. And then they can use this work to analyze and give the physical significance of various problems in the applied sciences.

6. Plan of Research work:

Year wise plan of work and targets to be achieved

S.N.	Period	Plan of Work	Target to be achieved
1	First Year	Collect the information through literature survey.	To Study of literature thoroughly and prepare subject.
2	Second Year	To analyze and collect research articles regarding relevant to research topic. Comparing the research articles.	With the help of research supervisor to prepare research article. To develop the idea of research article.
3	Third Year	Work on the some problems preparation of research articles.	Publish at least two research papers in journals.
4	Fourth Year	Publishing the research papers and submitting the thesis	Submitting the final synopsis. Thesis submission

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